Problem Set

Firm Dynamics and Productivity Growth

Please submit your answers as a single PDF file to pedro.armada@iwh-halle.de

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Problem 1. Hopenhayn-style firm dynamics

Consider a simplified version of the Hopenhayn model. Firms produce a homogeneous good using labor only. The product price is normalized to one, and the wage is $w > 0$. A firm with log productivity z produces:

$$y(z, n) = \exp(z)n^{\alpha}, \qquad 0 < \alpha < 1,$$

where n is labor demand. Log productivity evolves according to:

$$z' = \rho z + \varepsilon', \qquad 0 < \rho < 1, \qquad \varepsilon' \sim N(0, \sigma^2).$$

Firms pay a fixed operating cost $c_f > 0$. Labor is chosen after productivity is observed.

a) Write the firm's static profit maximization problem.

b) Derive optimal labor demand $n(z)$ and show that labor demand is increasing in productivity.

c) Using your expression for $n(z)$, derive the gross employment growth rate: $\frac{n(z')}{n(z)}$. Express it as a function of z, ρ, α, and ε'.

d) Compute the conditional expected growth rate: $\mathbb{E}\left[\frac{n(z')}{n(z)}\middle| z\right]$. Use the fact that, if $\varepsilon \sim N(0, \sigma^2)$, then $\mathbb{E}\left[\exp(a\varepsilon)\right] = \exp\left(\frac{a^2\sigma^2}{2}\right)$. Is expected growth increasing or decreasing in current productivity z?

e) Suppose, for this part only, that firms make a simple static exit decision after observing productivity: they operate if $\pi(z) - c_f \geq 0$ and exit otherwise. Derive the productivity threshold z^* below which firms exit.

f) Explain briefly how c_f and w affect the exit threshold z^*. Give the economic intuition.

g) In the full Hopenhayn model, firms exit based on continuation values, not only current profits. Explain why the exit rule still takes the form of a productivity threshold.

Problem 2. Current frontiers in firm dynamics

The canonical models studied in class show how firm dynamics can arise from selection, entry and exit, financial frictions, innovation, and other mechanisms. Recent work builds on these models by combining richer microdata with new sources of firm heterogeneity.

a) Identify one important feature of firm heterogeneity that is missing, or only partially captured, in the models discussed in class.

b) Explain why this source of heterogeneity matters. Through which channels could it affect aggregate outcomes?

c) Describe how you would extend one of the models studied in class to incorporate this source of heterogeneity.

d) What would be the ideal dataset to study this source of heterogeneity?

e) Suppose the ideal dataset is not available. What would be an indirect way to detect or measure this source of heterogeneity?

f) Conclude with one clear research question that could be studied using your proposed model and data.