Between Sector Misallocation

Pedro Armada*

September 2025

Preliminary and Incomplete.
Please do not cite or circulate.

Abstract

Structural change typically triggers shifts in sectoral composition with lasting effects on aggregate productivity, yet the underlying drivers can vary significantly across episodes, leading to different policy prescriptions. This paper develops a unified framework to quantify the contribution of sector-specific frictions in explaining sectoral reallocation and its macroeconomic implications. The model features rich heterogeneity at both the firm and sector levels, including differences in production technologies, competitive environments, risk exposure, and financial frictions. I apply the framework to Spain over the period 1992–2008, when capital inflows and falling interest rates following European integration coincided with a shift from tradable to nontradable sectors and a decline in total factor productivity. Using a simulated method of moments, I find that capital frictions–specifically, capital adjustment and sectoral mobility costs, estimated to be about 50% higher in the tradable sector–played a central role in the observed dynamics, highlighting the importance of policies aimed at easing capital frictions in order to improve allocative efficiency.

Keywords: misallocation, productivity, sectoral reallocation, capital frictions.

JEL codes: D24, E22, F41, O11.

*Halle Institute for Economic Research and Friedrich Schiller University Jena. pedro.armada@iwh-halle.de

1 Introduction

The allocation of economic activity across sectors plays a central role in shaping aggregate productivity dynamics, particularly during periods of structural transformation, trade liberalization, or credit booms. A salient example is the process of European integration between the Maastricht Treaty and the introduction of the euro, which triggered a convergence in interest rates among member countries and prompted large shifts in sectoral composition. During this period, activity reallocated from tradable to nontradable sectors, accompanied by a decline in aggregate TFP in the periphery and a growing productivity gap relative to the core. Understanding how sectoral reallocations interact with underlying frictions and structural differences is crucial for assessing their impact on aggregate productivity and for designing appropriate policy responses.

Differences in the emphasis placed on sectoral characteristics have led to markedly different interpretations of the sources of misallocation and, consequently, to divergent policy prescriptions. Studies emphasizing the insulation of nontradable sectors from international competition (Dias et al., 2016, 2020; Reis, 2013; Brunnermeier and Reis, 2023) advocate for pro-competition reforms to improve resource allocation. In contrast, work highlighting how capital inflows can crowd out high-productivity investments in tradable sectors (Benigno and Fornaro, 2014; Benigno et al., 2025) supports the use of capital controls or targeted sectoral subsidies to mitigate these effects. Other studies underscore tighter financial constraints in nontradable sectors stemming from weaker corporate governance, greater asset opacity, or heavier reliance on local credit markets (Tornell and Westermann, 2002; Schneider and Tornell, 2004; Ozhan, 2020), and point to reforms aimed at improving financial institutions.

Existing quantitative studies analyze the aggregate effects of capital inflows by developing models that emphasize particular frictions or sectoral differences. This approach carries the risk of over-attributing observed outcomes to the specific mechanisms embedded into the model by design. When only a narrow set of features is allowed to vary, the model is constrained to rationalize the data through those predetermined channels, potentially overstating their role or missing important interactions. This paper adopts a different approach. I develop a unified framework that incorporates a broad set of sectoral differences and frictions, and allow the model to internally weigh the relative importance of each mechanism and attribute the observed outcomes to the most relevant underlying characteristics.

To interpret the empirical patterns, I begin by extending the canonical misallocation framework of Hsieh and Klenow (2009) to a multi-sector setting that incorporates both sector-wide distortions and firm-level wedges. The extended framework highlights how sectoral asymmetries interact with firm dynamics, shaping both within-sector and between-sector resource allocation. I then present a simplified model that elucidates the endogenous sorting of firms across sectors and illustrates how capital inflows affect both the intensive and extensive margins of adjustment. This stylized model provides intuition for the quantitative analysis that follows.

Building on these insights, I develop a quantitative model with heterogeneous firms that features rich sectoral heterogeneity and a broad set of frictions. Sectors differ in their production technologies, competitive environment, exposure to idiosyncratic productivity shocks, and financial frictions–including the tightness of borrowing constraints, the magnitude of capital adjustment costs, and the degree of capital mobility across sectors.

To quantify the role of sectoral characteristics in accounting for the observed dynamics, I calibrate the model to the Spanish economy using a simulated method of moments approach. For each candidate set of parameters, I simulate the model's transition dynamics in response to the observed path of real interest rates from 1992 to 2008. The resulting model-implied time series for TFP, tradable output, and nontradable output are then compared to their empirical counterparts. The optimal parameters are those that minimize the distance between the simulated and observed series, allowing the data to discipline the relative importance of the different sectoral characteristics.

The calibrated model emphasizes capital frictions as central to explaining the observed dynamics. Both capital adjustment costs and sector-switching costs are estimated to be about 50% higher in the tradable sector than in the nontradable sector. Intuitively, these frictions limit the tradable sector's response to falling interest rates on two fronts: on the intensive margin, capital adjustment costs constrain investment by incumbent firms, and on the extensive margin, switching costs discourage entry by nontradable firms, further slowing the expansion. Although the model successfully replicates the TFP loss and the reallocation of activity from tradables to nontradables observed during this period, it underestimates the magnitude of the nontradable sector's expansion, suggesting that additional forces beyond the modeled frictions and sectoral asymmetries may have played an important role during the transition.

From a policy perspective, the results highlight the importance of capital adjustment frictions in hindering the reallocation of productive resources across sectors. This suggests that policy efforts should not be limited to promoting competition or access to credit but should also focus on easing capital mobility. Policies aimed at developing secondary markets for capital goods and facilitating the transfer or sale of businesses appear promising in improving productivity.

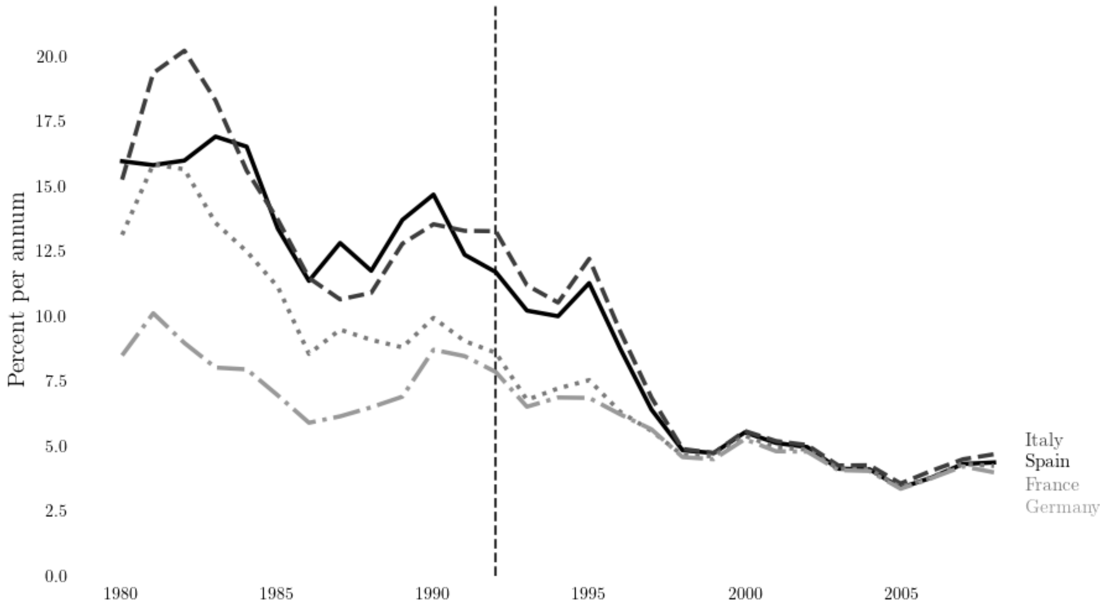
The remainder of the paper is organized as follows. Section 2 presents motivating evidence on sectoral reallocation and productivity dynamics in Europe around the time of the Maastricht Treaty. Section 3 introduces a multi-sector conceptual framework to illustrate how resource misallocation affects aggregate productivity. Section 4 outlines a simplified model to demonstrate the effect of firm-level and sectoral distortions on resource allocation. Section 5 develops a quantitative model featuring heterogeneous firms and sectors. Section 6 describes the calibration strategy and presents the results from the quantitative analysis. Section 7 concludes.

2 Motivating Evidence

This section documents the evolution of total factor productivity (TFP) and the sectoral dynamics in Europe that took place during the period surrounding the implementation of the Maastricht Treaty. I will restrict attention to Spain, Italy, Germany, and France, which together accounted for approximately 70% of the total GDP among the original signatory members of the Treaty. This selection is driven by data availability across multiple sources used in this section. Moreover, these four countries are emblematic of the different experiences of core and periphery economies in the transition toward monetary union.

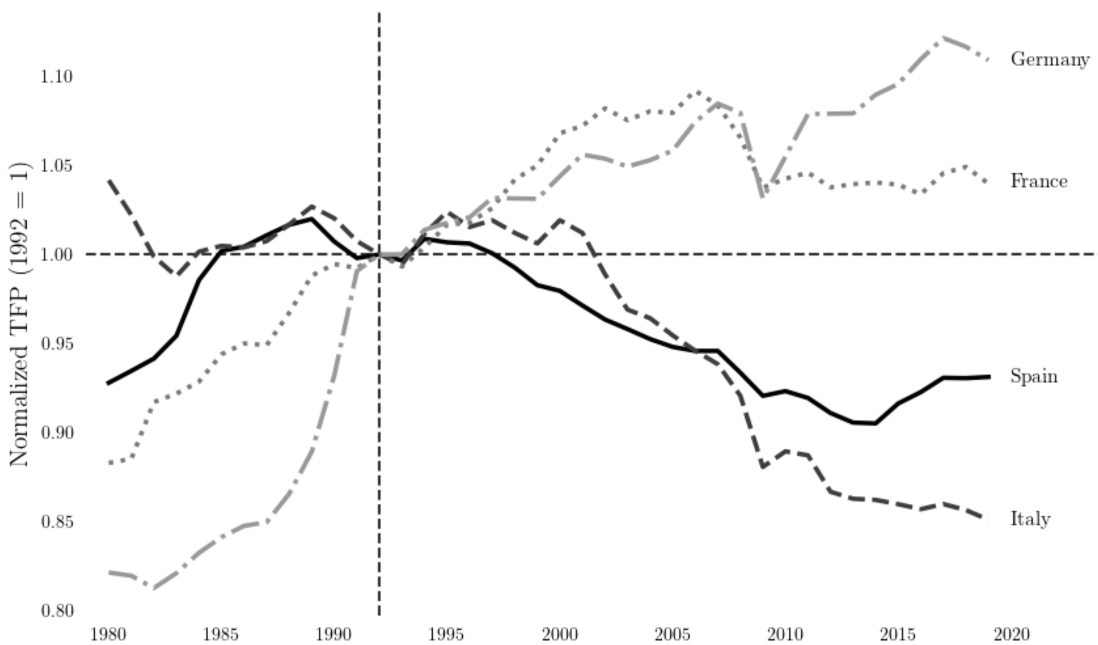
The Maastricht Treaty, signed in 1992, laid the foundation for the creation of the European Union and set the framework for the introduction of a single European currency, after decades of fragmented monetary policies and fluctuating exchange

Figure 1. Interest rates on government bonds



Notes: This figure shows the evolution of government bond yields from 1980 to 2008 using data from the International Financial Statistics, produced by the International Monetary Fund. The dashed vertical line indicates 1992, the year of the Maastricht Treaty.

Figure 2. Aggregate Productivity



Notes: This figure shows the evolution of Total Factor Productivity (TFP) at constant national prices from 1980 to 2019 using data from the Penn World Table, version 10.01 (Feenstra et al., 2015). TFP is normalized to 1 in 1992, the year of the Maastricht Treaty (dashed vertical line).

rates. A key motivation behind the Maastricht Treaty was to foster economic convergence among European economies by establishing a common set of macroeconomic criteria. These criteria included limits on inflation rates, government debt, budget deficits, exchange rate volatility, and long-term interest rates.

One of the most striking effects of the Maastricht Treaty was the rapid convergence of interest rates across European economies in the years leading up to the adoption of the euro in 1999. Prior to the treaty, interest rates varied greatly across member states, reflecting differences in inflation expectations, fiscal discipline, and perceived sovereign risk. As shown in Figure 2, countries with histories of high inflation and weaker fiscal positions, such as Spain and Italy, faced substantially higher borrowing costs compared to economies like Germany and France. In 1992, the interest rate differential between Italy and Germany stood at 540 basis points, while the gap between Spain and Germany was 380 basis points. By 1999, these differentials had narrowed to less than 25 basis points. This sharp convergence in government bond yields translated into a decline in corporate borrowing rates, since the cost of borrowing for corporate borrowers is closely tied to the yields that sovereigns pay on their debt (Eichengreen and Mody, 2000; Durbin and Ng, 2005; Bedendo and Colla, 2015; Bevilaqua et al., 2020).

By reducing interest rate differentials and eliminating exchange rate uncertainty, the Treaty was expected to foster capital deepening, encourage investment, and improve resource allocation. However, the extent to which these changes translated into gains in aggregate productivity varied significantly across countries (Reis, 2013; Benigno and Fornaro, 2014; Cette et al., 2016; Gopinath et al., 2017; García-Santana et al., 2020; Brunnermeier and Reis, 2023). As shown in Figure 2, TFP trajectories diverged following the Treaty, with Germany and France experiencing productivity gains, while Spain and Italy saw persistent declines. More than a quarter-century later, these disparities remain stark. Compared to 1992, Spain's TFP declined by 7% and Italy's by 15% by 2019. In contrast, Germany saw an 11% increase in TFP, and France experienced a 4% rise over the same period.

This period coincided with a rapid expansion of the nontradable sector across European countries, particularly in Spain and Italy. As shown in Figure 3, aggregate output in nontradable sectors increased across all four economies. However, their trajectories relative to pre-Treaty trends reveal divergent paths. While France and Germany maintained growth in nontradable output broadly in line with their pre-Treaty trends, Spain and Italy experienced a marked acceleration following the Treaty's implementation. By 2008, nontradable output had grown 176% above its pre-Treaty trend in Spain and 45% in Italy, compared to 20% in France and 9% in Germany. In contrast, as displayed in Figure 4, output in the tradable sector also increased during this period, but with a notable deceleration relative to trend in Germany, France, and Italy, and only a modest acceleration in Spain.

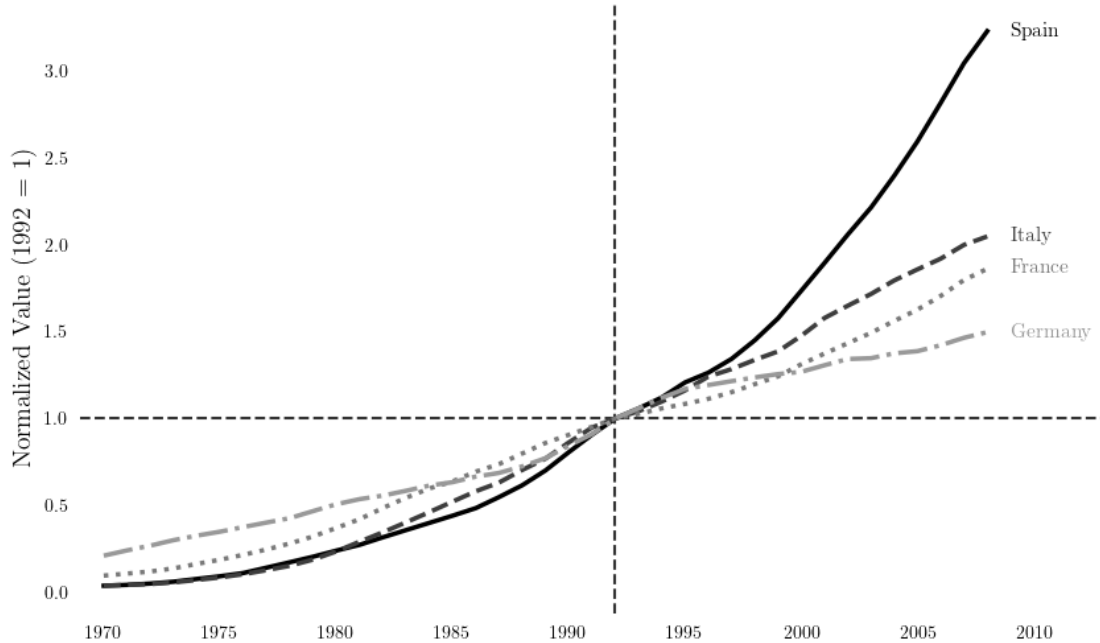
These sectoral dynamics are consistent with an extensive literature documenting the effects of capital inflow episodes across a broad set of countries and historical periods (Tornell and Westermann, 2002; Benigno et al., 2015; Kalantzis, 2015; Müller and Verner, 2024).

To interpret these patterns, I now develop a framework that links capital inflows to changes in sectoral composition and aggregate productivity, designed to rationalize the key empirical facts documented above: a decline in interest rates, a reallocation of activity away from tradables and toward nontradables, and a deterioration in allocative efficiency.

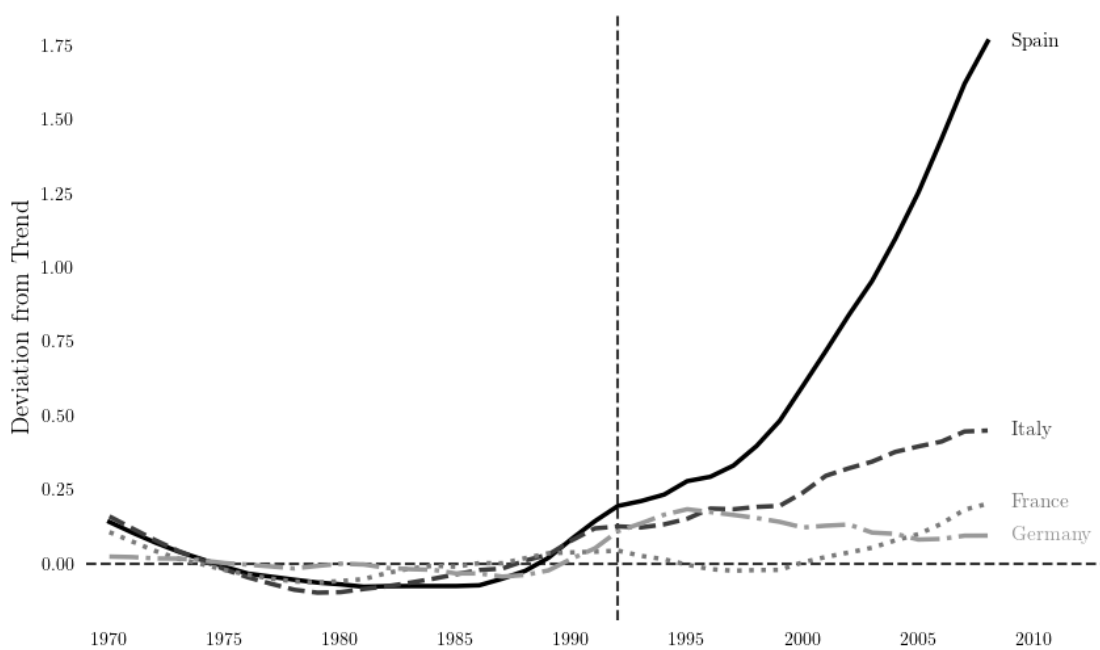
3 Conceptual Framework

This section builds on the framework of Hsieh and Klenow (2009) which models heterogeneous firms engaged in monopolistic competition to analyze the impact of resource misallocation on aggregate productivity. Beyond firm-level output and capital distortions that affect the allocation of resources within sectors, this framework incorporates sectoral heterogeneity and sector-wide distortions that propagate to all firms within a given sector. Importantly, sectoral distortions interact with firm-level distortions, influencing both individual firm decisions and the allocation of resources across sectors.

Figure 3. Nontradable Sector: Output and Deviation from Trend



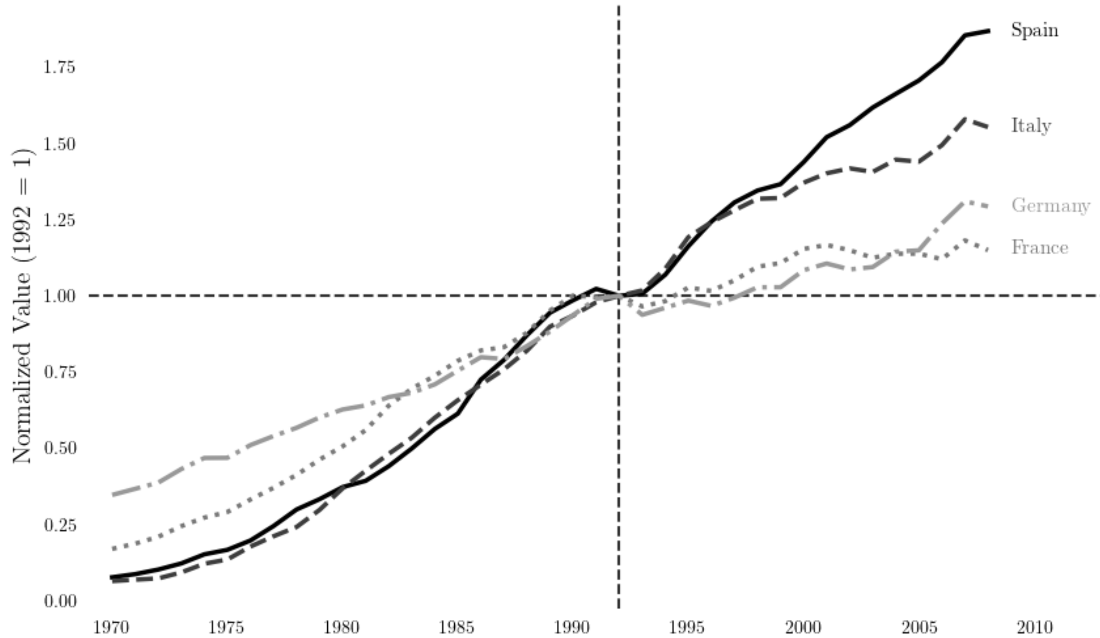
(a) Gross Output in Nontradable Sectors (Normalized, 1992 = 1)



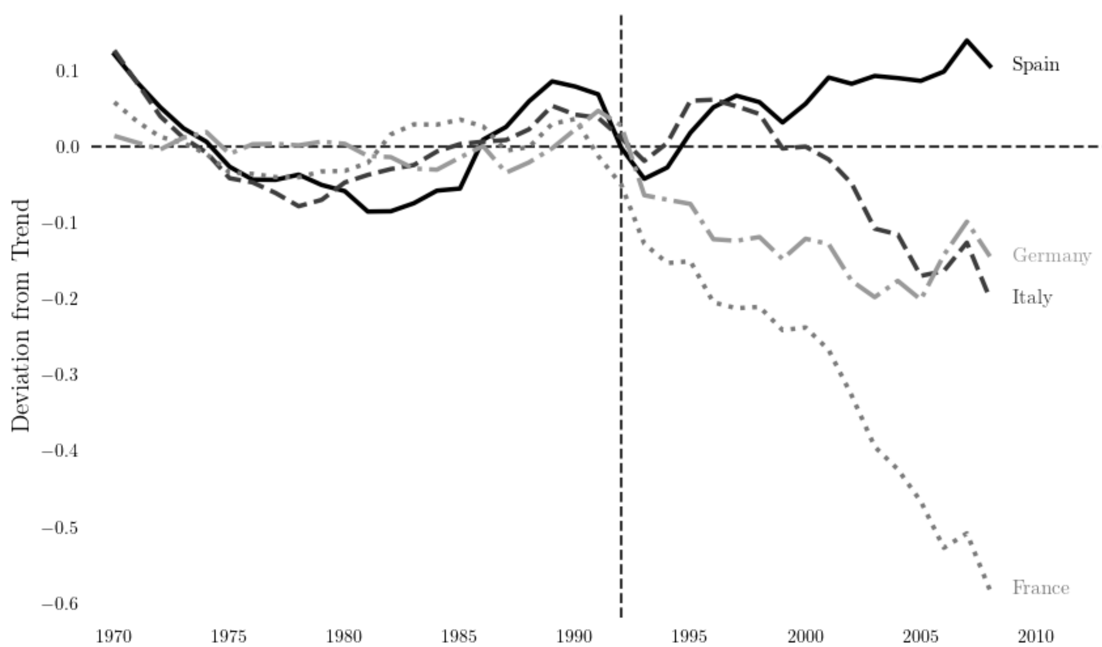
(b) Deviation from Trend Output in Nontradable Sectors

Notes: Data are from the historical EU KLEMS (October 2012 release) (O'Mahony and Timmer, 2009). The tradable sector includes primary industries and manufacturing. All remaining sectors are classified as nontradable. Output is normalized to 1 in 1992, the year of the Maastricht Treaty (dashed vertical line). Trend deviations are calculated using country-specific linear trends from 1970 to 1992.

Figure 4. Tradable Sector: Output and Deviation from Trend



(a) Gross Output in Tradable Sectors (Normalized, 1992 = 1)



(b) Deviation from Trend Output in Tradable Sectors

Notes: Data are from the historical EU KLEMS (October 2012 release) (O'Mahony and Timmer, 2009). The tradable sector includes primary industries and manufacturing. All remaining sectors are classified as nontradable. Output is normalized to 1 in 1992, the year of the Maastricht Treaty (dashed vertical line). Trend deviations are calculated using country-specific linear trends from 1970 to 1992.

3.1 Setup

The final good Y_t is a constant elasticity of substitution (CES) aggregate of sectoral outputs Y_{st} produced across S sectors:

$$Y_t = \left(\sum_{s=1}^{S} Y_{st}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{1}$$

where $\eta > 1$ denotes the elasticity of substitution across sectoral outputs. This CES specification represents a departure from the Cobb-Douglas aggregation in Hsieh and Klenow (2009), which implies that changes in sectoral productivity have no effect on the allocation of inputs across sectors. With a Cobb-Douglas aggregation, a rise in sectoral productivity leads to a proportional fall in that sector's price index, leaving expenditure shares and input allocations across sectors unchanged. Since the focus of this paper is on intersectoral allocation, a CES aggregator is necessary to allow sectoral distortions and productivity differences to influence aggregate outcomes through their effect on the composition of output.

Letting P_{st} denote the sectoral price index, the aggregate price index P_t of the final good is given by:

$$P_t = \left(\sum_{s=1}^{S} P_{st}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{2}$$

Each sectoral output Y_{st} is itself a CES aggregate of differentiated products produced by N_{st} individual firms:

$$Y_{st} = \left(\sum_{i=1}^{N_{st}} y_{ist}^{\frac{\theta_s-1}{\theta_s}} \right)^{\frac{\theta_s}{\theta_s-1}} \tag{3}$$

which follows the formulation in Hsieh and Klenow (2009) but allows for sector-specific elasticities of substitution θ_s. The parameter θ_s governs the degree of substitutability between different varieties within sector s, with a higher θ_s implying that products within the sector are more easily substitutable. As a result, firms within sector s face an isoelastic demand function:

$$y_{ist} = \left(\frac{p_{ist}}{P_{st}}\right)^{-\theta_s} \left(\frac{P_{st}}{P_t}\right)^{-\eta} Y_t \tag{4}$$

where p_{ist} denotes the price of variety i and P_{st} is the sectoral price index, defined as:

$$P_{st} = \left(\sum_{i=1}^{N_{st}} p_{ist}^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \tag{5}$$

Finally, the production function for each differentiated product follows a Cobb-Douglas specification:

$$y_{ist} = z_{ist} k_{ist}^{\alpha_s} l_{ist}^{1-\alpha_s} \tag{6}$$

where k_{ist} and l_{ist} represent capital and labor inputs, respectively, z_{ist} denotes firm-level total factor productivity, and α_s is a sector-specific output elasticity of capital.

3.2 Wedges

Firm i in sector s chooses its price, capital, and labor to maximize profits:

$$\max_{p_{ist}, k_{ist}, l_{ist}} \pi_{ist} = (1 - \tilde{\tau}_{ist}^y) p_{ist} y_{ist} - (1 + \tilde{\tau}_{ist}^k)(r_t + \delta_s) k_{ist} - w_{st} l_{ist} \tag{7}$$

where w_{st} is the wage rate in sector s, r_t the real interest rate, and δ_s the depreciation rate of capital in sector s. The terms $\tilde{\tau}_{ist}^y$ and $\tilde{\tau}_{ist}^k$ represent composites wedges, capturing both firm-specific and sectoral distortions that affect firms' decisions: $\tilde{\tau}_{ist}^y$ distorts output and $\tilde{\tau}_{ist}^k$ distorts capital relative to labor. Wages w_{st} may vary across sectors due to, for example, differences in human capital or labor mobility costs. Similarly, capital depreciation rates δ_s may differ across sectors because of variations in capital composition, intensity of use, or sector-specific technological characteristics. Although the real interest rate r_t is common across all firms, the effective cost of capital may vary due to both firm-specific and sector-wide capital distortions. These wedges are treated as exogenous for now but are endogeneized in the quantitative model in Section 5.

The first-order conditions with respect to labor and capital are given by:

$$MRPL_{ist} = \left(\frac{1-\alpha_s}{\mu_s}\right)\left(\frac{p_{ist}y_{ist}}{l_{ist}}\right) = \left(\frac{1}{1-\tilde{\tau}^y_{ist}}\right) w_{st} \tag{8}$$

$$MRPK_{ist} = \left(\frac{\alpha_s}{\mu_s}\right)\left(\frac{p_{ist}y_{ist}}{k_{ist}}\right) = \left(\frac{1+\tilde{\tau}^k_{ist}}{1-\tilde{\tau}^y_{ist}}\right)(r_t+\delta_s) \tag{9}$$

where $\mu_s = \frac{\theta_s}{\theta_s - 1}$ denotes the constant markup of price over marginal cost in sector s, under the assumption that each firm is small relative to its sector and thus takes the sectoral price index P_{st} as given.

The total revenue-based total factor productivity (TFPR) at the firm level is defined as:

$$TFPR_{ist} = p_{ist}z_{ist} = \frac{p_{ist}y_{ist}}{k_{ist}^{\alpha_s} l_{ist}^{1-\alpha_s}} = \mu_s \left(\frac{MRPK_{ist}}{\alpha_s}\right)^{\alpha_s}\left(\frac{MRPL_{ist}}{1-\alpha_s}\right)^{1-\alpha_s} \tag{10}$$

showing that positive output distortions $\tilde{\tau}^y_{ist}$ or positive capital distortions relative to labor $\tilde{\tau}^k_{ist}$ lead to higher marginal revenue products.

In an efficient economy without distortions, the marginal revenue products of labor and capital would be equalized across firms. However, in the presence of distortions, firms equate the marginal revenue product of each input to its distorted marginal cost. As a result, the allocation of resources across firms is influenced not only by firm-level total factor productivity but also by the output and capital distortions they face.

As in Hsieh and Klenow (2009), when production relies on two factors of production, it is possible to distinguish distortions that affect both capital and labor from those that alter the relative marginal product of one factor compared to the other. Output distortions τ_Y act as frictions that proportionally raise the marginal products of both capital and labor. These distortions act as implicit taxes or subsidies on firm output. For instance, firms facing government-imposed size restrictions, high transportation costs, or bureaucratic barriers experience high τ_Y, while firms benefiting from preferential treatment, such as public subsidies or regulatory exemptions, face low τ_Y. Output distortions directly impact firm-level production by restricting their ability to expand, independent of input factor choices. In contrast, capital distortions

τ_K affect the relative cost of capital compared to labor, altering the optimal input mix. Firms that face credit constraints, such as those operating in financial markets with limited access to loans or high borrowing costs, encounter high τ_K, leading them to substitute away from capital-intensive production. Conversely, firms benefiting from cheap credit – whether due to state-backed lending programs, business group affiliations, or implicit government guarantees – experience low τ_K, enabling them to over-accumulate capital relative to labor.

In addition to firm-specific distortions highlighted by Hsieh and Klenow (2009), an additional layer of inefficiencies arises due to sectoral distortions: structural frictions that affect all firms in the same sector. These distortions may arise from industrial policies, differential tax policies, sector-specific credit allocation, regulatory barriers, or market concentration effects. Unlike firm-level distortions, which introduce dispersion of marginal products within sectors, sectoral distortions create systematic imbalances in marginal products across sectors. In economies with sectoral distortions, some industries may operate below their efficient scale, while others may expand inefficiently, absorbing disproportionate resources without generating commensurate productivity gains.

The composite distortions affecting output and capital reflect the combined impact of micro-level and sector-wide distortions on firms' production decisions. These composite wedges are given by:

$$(1 - \tilde{\tau}^y_{ist}) = (1 - \tau^y_{ist})(1 - \tau^y_{st}) \tag{11}$$

$$(1 + \tilde{\tau}^k_{ist}) = (1 + \tau^k_{ist})(1 + \tau^k_{st}) \tag{12}$$

where τ^y_{ist} and τ^k_{ist} represent firm-specific wedges, whereas τ^y_{st} and τ^k_{st} represent sector-wide wedges. A firm experiencing both a sector-wide distortion and a firm-specific distortion faces an amplified misallocation effect, as the total wedge reflects the compounding impact of these frictions. When sectoral distortions are absent (i.e., $\tau^y_{st} = 0$ and $\tau^k_{st} = 0$), the composite wedge collapses to the standard firm-specific wedges τ^y_{ist} and τ^k_{ist}, as in Hsieh and Klenow (2009).

3.3 Aggregation

We are now ready to introduce measures of productivity and misallocation at both the sectoral and aggregate levels. Total capital and labor employed in sector s at time t are given by:

$$K_{st} = \sum_{i=1}^{N_{st}} k_{ist} \qquad \text{and} \qquad L_{st} = \sum_{i=1}^{N_{st}} l_{ist}. \tag{13}$$

Total factor productivity (TFP) at the sector level is defined as:

$$TFP_{st} = \frac{Y_{st}}{K_{st}^{\alpha_s} L_{st}^{1-\alpha_s}} \tag{14}$$

which, using the sectoral price index $P_{st} = \left(\sum_{i=1}^{N_{st}} p_{ist}^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}}$ and $p_{ist} = \frac{TFPR_{ist}}{z_{ist}}$, can be expressed as:

$$TFP_{st} = \left(\sum_{i=1}^{N_{st}} \left(z_{ist} \frac{TFPR_{st}}{TFPR_{ist}}\right)^{\theta_s - 1}\right)^{\frac{1}{\theta_s - 1}} \tag{15}$$

In a frictionless economy without distortions, $TFPR_{ist} = TFPR_{st}$ across all firms, and sectoral TFP reduces to:

$$TFP_{st}^{e} = \left(\sum_{i=1}^{N_{st}} z_{ist}^{\theta_s - 1}\right)^{\frac{1}{\theta_s - 1}} \tag{16}$$

The presence of distortions introduces dispersion in firm-level TFPR, leading to a loss in sectoral productivity. The loss in sectoral TFP due to misallocation is given by:

$$\log\left(\frac{TFP_{st}}{TFP_{st}^{e}}\right) = \frac{1}{\theta_s - 1}\left[\log\left(\sum_i z_{ist}^{\theta_s - 1}\left(\frac{TFPR_{st}}{TFPR_{ist}}\right)^{\theta_s - 1}\right) - \log\left(\sum_i z_{ist}^{\theta_s - 1}\right)\right] \tag{17}$$

Aggregate capital, labor, and output across sectors are:

$$K_t = \sum_{s=1}^{S} K_{st} \qquad \text{and} \qquad L_t = \sum_{s=1}^{S} L_{st} \qquad \text{and} \qquad Y_t = \left(\sum_{s=1}^{S} Y_{st}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{18}$$

Substituting the production structure, aggregate output can also be written as:

$$Y_t = \sum_{s=1}^{S} \left(TFP_{st}^{\frac{\eta-1}{\eta}} \cdot K_{st}^{\frac{\alpha_s(\eta-1)}{\eta}} \cdot L_{st}^{\frac{(1-\alpha_s)(\eta-1)}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{19}$$

Finally, aggregate TFP is given by:

$$TFP_t = \left(\sum_{s=1}^{S} \omega_{st} \; TFP_{st}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{20}$$

where sectoral weights ω_{st} are determined by relative sectoral output $\omega_{st} = \frac{Y_{st}^{\frac{\eta-1}{\eta}}}{\sum_{s'=1}^{S} Y_{s't}^{\frac{\eta-1}{\eta}}}$.

4 Illustrative Model

To develop intuition for the results derived in the quantitative model of Section 5, I first present a simplified framework that illustrates the impact of firm-level and sectoral distortions on resource allocation. This model elucidates how sectors attract different types of firms and how distortions influence firm sorting across sectors. Additionally, it provides insight into the dynamics of within-sector and between-sector misallocation induced by the convergence in interest rates, which is the main driving force in the quantitative model.

4.1 Environment

Consider a static economy where firms differ in their idiosyncratic productivity z and operate a linear technology with capital k as the sole factor of production:

$$y = zk \tag{21}$$

In turn, sectors vary in their demand elasticity θ_s. Firms compete in monopolistically competitive markets, each producing a differentiated variety within its sector. The demand for each firm's variety is determined by:

$$y = p^{-\theta_s} \tag{22}$$

where p denotes the price of the firm's variety.

Before production, each firm makes a once-and-for-all decision regarding which sector s to enter by comparing expected profits across sectors:

$$s^\star = \arg\max_s \ \mathbb{E}\left[\pi(s)\right] \tag{23}$$

Once a firm enters sector s, it chooses its capital input to maximize its expected profit:

$$\pi(s) = \max_k \ \mathbb{E}\left[(zk)^{\frac{\theta_s - 1}{\theta_s}} - (1 + \tilde{\tau}^k) r k\right] \tag{24}$$

where $\tilde{\tau}^k$ is a stochastic capital distortion and r is the cost of capital. Importantly, the firm chooses inputs before observing realized capital distortions.

Theorem 4.1. *The optimal capital choice that maximizes expected profit is given by:*

$$k^* = \frac{z^{\theta_s - 1}}{\mu_s^{\theta_s} r^{\theta_s} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s}} \tag{25}$$

Proof. See Appendix Appendix A.1. □

As in Section 3, capital distortions consist of both a firm-level wedge and a sectoral wedge, which are assumed to be correlated. The composite distortion is given by:

$$(1 + \tilde{\tau}^k) = (1 + \tau_i^k)(1 + \tau_s^k) \tag{26}$$

where τ_i^k represents the firm-specific distortion, while τ_s^k captures distortions that apply uniformly across all firms within sector s.

Theorem 4.2. *The expected capital distortion is given by:*

$$\mathbb{E}(1+\tilde{\tau}^k) = 1 + \mathbb{E}(\tau_i^k) + \mathbb{E}(\tau_s^k) + \mathbb{E}(\tau_i^k)\mathbb{E}(\tau_s^k) + Cov(\tau_i^k, \tau_s^k) \tag{27}$$

Proof. See Appendix Appendix A.2. □

Expected capital distortions depend on expected firm-level and sectoral distortions, their interaction, and their covariance. If firm- and sectoral-level wedges are positively correlated, the covariance term is positive, amplifying the total effect. If they are negatively correlated, it mitigates the total expected distortion.

4.2 Effect on Input Choice

To see the impact of distortions on the firm's input choice, it is instructive to contrast it with the efficient benchmark, which assumes no capital distortions.

Theorem 4.3. *The efficient level of capital in the absence of distortions is given by:*

$$k^e = \frac{z^{\theta_s - 1}}{\mu_s^{\theta_s} r^{\theta_s}} \tag{28}$$

Proof. See Appendix Appendix A.3. □

The ratio between the optimal choice k^* relative to its efficient counterpart k^e is therefore:

$$\frac{k^*}{k^e} = \left(\frac{1}{\mathbb{E}(1+\tilde{\tau}^k)}\right)^{\theta_s} = \left(\frac{1}{1 + \mathbb{E}(\tau_i^k) + \mathbb{E}(\tau_s^k) + \mathbb{E}(\tau_i^k)\mathbb{E}(\tau_s^k) + \mathrm{Cov}(\tau_i^k, \tau_s^k)}\right)^{\theta_s} \tag{29}$$

which is less than unity, reflecting the fact that distortions depress capital accumulation. When expected distortions are large (i.e., $\mathbb{E}(1+\tilde{\tau}^k)$ is high), firms operate with less capital compared to the efficient benchmark. If firm-specific distortions are high, the firm reduces investment, even if the sector as a whole is not constrained. If the entire sector faces distortions, all firms in that sector reduce investment.

The extent to which firm-specific and sector-wide distortions are correlated also influences resource allocation both within and between sectors. If firm-specific and sectoral distortions are positively correlated (i.e., firms with high idiosyncratic distortions tend to be in highly distorted sectors), the distortionary effect is amplified. In the extreme case where firm- and sector-level distortions are perfectly correlated ($\tau_s^k = \tau_i^k$), the total distortion becomes quadratic, with the capital choice relative to the efficient benchmark given by:

$$\frac{k^*}{k^e} = \left(\frac{1}{1 + 2\mathbb{E}(\tau_i^k) + \mathbb{E}(\tau_i^k)^2 + \text{Var}(\tau_i^k)} \right)^{\theta_s} \tag{30}$$

Conversely, when firm- and sector-level distortions are negatively correlated, sector-wide distortions partially offset firm-level frictions. In this scenario, firms facing higher firm-level distortions experience a sector-level subsidy that reduces the effective capital wedge. In the special case of perfectly negative correlation ($\tau_s^k = -\tau_i^k$), the distortion ratio equals one, and efficiency is restored.

Moreover, the extent to which the optimal capital choice deviates from the efficient benchmark is influenced by the sectoral demand elasticity (θ_s).

Theorem 4.4. *Distortions lead to a greater reduction in capital accumulation in sectors with a higher elasticity of substitution.*

Proof. See Appendix Appendix A.4. □

This follows from the derivative:

$$\frac{\partial(k^*/k^e)}{\partial\theta_s} = -\left(\mathbb{E}(1+\tilde{\tau}^k)\right)^{-\theta_s} \log\left(\mathbb{E}(1+\tilde{\tau}^k)\right) \tag{31}$$

which is negative, since $\mathbb{E}(1+\tilde{\tau}^k) > 0$ for positive distortions, making $\log \mathbb{E}(1+\tilde{\tau}^k)$ positive. This means that as θ_s increases, the ratio k^*/k^e decreases, implying a larger deviation from the efficient benchmark.

Intuitively, in high-elasticity sectors, firms have lower market power and cannot easily pass higher capital costs onto consumers. As a result, the effect of capital distortions is amplified, leading to an even greater reduction in capital. In contrast, in low-elasticity sectors, firms command greater pricing power, allowing them to absorb distortions by adjusting prices, thereby mitigating the impact on capital accumulation.

4.3 Effect on Firm Sorting

Consider next the effect of capital wedges on how firms sort across sectors.

Theorem 4.5. *Given the optimal capital choice k^*, the expected profit of a firm operating in sector s, subject to capital distortions τ^k, is:*

$$\pi(s) = \frac{(\theta_s - 1)^{\theta_s - 1} z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s - 1} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s - 1}} \tag{32}$$

Proof. See Appendix Appendix A.5. □

Before showing how firms sort into sectors according to their productivity, it is useful to highlight that sectoral elasticity determines how profits scale with productivity.

Theorem 4.6. *The elasticity of optimal profits with respect to productivity is determined solely by the sectoral demand elasticity θ_s:*

$$\frac{\partial \ln \pi(s)}{\partial \ln z} = \theta_s - 1 \tag{33}$$

Thus, productivity gains translate more strongly into profit gains when θ_s is large. Intuitively, in sectors with more elastic demand (i.e., higher θ_s), profits increase more steeply with productivity. As a result, high-productivity firms are drawn to high-elasticity sectors, where they can benefit from a larger scale. In contrast, sectors with less elastic demand offer higher markups, making them more attractive to low-productivity firms that benefit from wider profit margins.

In fact, the demand elasticity has a positive impact on the firm's profit as long as its productivity is above the markup-adjusted cost of capital, as can be seen by the sign of the elasticity of optimal profits with respect to demand elasticity.

Theorem 4.7. *The elasticity of optimal profit with respect to the sectoral demand elasticity θ_s is given by:*

$$\frac{\partial \ln \pi(s)}{\partial \ln \theta_s} = \ln\left(\frac{z}{\mu_s \, r}\right) \tag{34}$$

which is positive if and only if:

$$z > \mu_s \, r \tag{35}$$

Firms will sort into sectors depending on their productivity, the cost of capital, differences in sector elasticity, as well as firm-specific and sector-wide capital wedges.

Theorem 4.8. *A firm choosing between two sectors, A and B, which differ in their elasticity of substitution, with $\theta_A > \theta_B > 1$, strictly prefers to operate in sector A over sector B if its productivity z satisfies:*

$$z > \underbrace{r}_{\text{Cost of Capital}} \cdot \underbrace{\left(\frac{\theta_B - 1}{\theta_A - 1} \cdot \frac{\mu_A^{\theta_A}}{\mu_B^{\theta_B}} \right)^{\frac{1}{\theta_A - \theta_B}}}_{\text{Gains from Scale vs. Markups}} \cdot \underbrace{\frac{\mathbb{E}(1 + \tilde{\tau}^k(A))^{\frac{\theta_A - 1}{\theta_A - \theta_B}}}{\mathbb{E}(1 + \tilde{\tau}^k(B))^{\frac{\theta_B - 1}{\theta_A - \theta_B}}}}_{\text{Capital Distortions}} \tag{36}$$

Proof. See Appendix Appendix A.6. □

The expression for the threshold productivity level can be broken down into three components. The first term, r, reflects the cost of capital. As the interest rate increases, so does the threshold productivity, such that only more productive firms will find it worthwhile to operate in the high-elasticity sector.

The second term captures the trade-off between scale gains and markups across sectors with different elasticities. This term adjusts the threshold by comparing how steeply profits rise with productivity against the relative markup levels. High-elasticity sectors reward productivity more with bigger scale gains, whereas low-elasticity sectors offer higher margins and are more attractive to low-productivity firms.

The third term reflects the relative tightness of capital distortions across sectors. If sector A exhibits higher capital distortions, it increases the productivity threshold above which firms choose to operate in it. It also captures the distortion induced by capital wedges, since the marginal firm that would otherwise operate in the high elasticity sector will choose to operate in the alternative sector.

4.4 Effect of Capital Inflows

Capital inflows, modeled as a decline in the cost of capital r, affect both the intensive margin (input choice) and the extensive margin (sector choice) of firm behavior.

On the intensive margin, it is straightforward to show that capital is more responsive to interest rate changes in high elasticity sectors. The responsiveness of capital to changes in the cost of capital is captured by the elasticity:

$$\frac{\partial \ln k^*}{\partial \ln r} = -\theta_s \tag{37}$$

which is negative, indicating that a given decline in r induces a stronger expansion of capital in sectors with more elastic demand (i.e., higher θ_s).

On the extensive margin, a decline in the cost of capital r lowers the productivity threshold above which firms choose to operate in the high-elasticity sector. As shown in Equation 36, when comparing two sectors that differ in their elasticity of substitution, the threshold productivity required to prefer the high-elasticity sector declines with r. Consequently, capital inflows induce a re-sorting of firms across sectors, with marginally productive firms shifting toward sectors with more elastic demand.

5 Quantitative Model

This section develops a multi-sector model to evaluate how capital and sectoral reallocation impact aggregate total factor productivity (TFP). The model builds on the quantitative framework of Gopinath et al. (2017), incorporating rich sectoral heterogeneity to capture differences in production technology, market power, and financial frictions across sectors. Compared to the theoretical framework outlined in Section 4, the quantitative model allows for evaluating the merits of competing channels in explaining the observed dynamics, as well as quantifying their relative importance.

5.1 Primitives

I consider a small open economy populated by a large number of monopolistically competitive firms, each deciding whether to operate in the tradable (T) or nontradable (N) sector. Each firm i operating in sector $s = \{T, N\}$ at time t is owned by a risk-averse entrepreneur who chooses consumption c_{ist} to maximize the expected value of the sum of discounted utility flows:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{ist}^{1-\gamma} - 1}{1-\gamma} \right) \tag{38}$$

where β denotes the discount factor and γ the coefficient of relative risk aversion.

Firms hire labor competitively at the prevailing wage w_t, which is taken as given. They can also save and borrow in the international credit market through a one-period bond that trades at an exogenous real interest rate r_t. There is no aggregate uncertainty.

5.2 Production Technology

Firms produce physical output y_{ist} using a Cobb-Douglas production function given by:

$$y_{ist} = z_{ist} k_{ist}^{\alpha_s} l_{ist}^{1-\alpha_s} \tag{39}$$

where z_{ist} is firm productivity, k_{ist} is the capital stock, and l_{ist} is labor input. The parameter $\alpha_s \in (0, 1)$ determines the sector-specific capital share.

Differences in capital shares stem from variations in factor intensity across sectors. In capital-intensive sectors, such as manufacturing, firms rely more on physical capital (machinery, equipment, and infrastructure) relative to labor, leading to a higher capital share. In contrast, labor-intensive sectors, such as services, depend more on human input and require less capital, resulting in a lower capital share.[1]

[1] For example, Valentinyi and Herrendorf (2008) find that capital shares differ across sectors, with higher capital intensity in agriculture (0.54) and manufacturing (0.40) compared to services (0.34) and construction (0.21). Relevant to my analysis, their findings indicate that the capital share in the tradable sector is approximately 0.37, while in the nontradable sector, it is slightly lower at 0.32.

Given the prevailing wage w_t, the firm's operating profits are given by:

$$\pi_{ist} = p_{ist}y_{ist} - w_t l_{ist} - \kappa_s \tag{40}$$

where p_{ist} is the price of the firm's variety and $\kappa_s \geq 0$ represents a sector-specific fixed cost. Sectoral differences in fixed operating costs may arise due to variations in infrastructure requirements, regulatory compliance, and technological setup costs. As in Buera et al. (2011), the presence of fixed operating costs introduces a non-convexity in firms' production decisions, leading to differences in the scale of operation across sectors.

5.3 Market Structure

Firms operate in a monopolistically competitive environment, where each firm is the sole supplier of a differentiated variety. Consistent with the framework outlined in Section 3, sectoral output is a CES aggregate of differentiated products. As a result, firms face downward-sloping demand for their individual varieties, given by:

$$y_{ist} = p_{ist}^{-\theta_s} \tag{41}$$

where p_{ist} is the price of the differentiated varieties and θ_s is the sector-specific absolute value of the elasticity of demand. Following Gopinath et al. (2017), I abstract from the determination of the sectoral price index and normalize both the demand shifter and the sectoral price index to 1 in the demand function. Given the CES demand structure, firms charge uniform markups within each sector:

$$\mu_s = \frac{\theta_s}{\theta_s - 1} \tag{42}$$

with differences in markups across firms arising solely from differences in sectoral elasticity. The sectoral markup is decreasing in θ_s, meaning that markups are lower in sectors with higher elasticity of substitution, and vice-versa.

Sectoral differences in demand elasticity may arise due to varying degrees of exposure to competition. In highly competitive sectors (e.g., manufacturing), firms face pressures from both domestic and international competitors, providing consumers with abundant substitutes. This makes demand more elastic and restricts firms' ability to charge prices significantly above marginal costs. Conversely, in less competitive sectors (e.g., real estate), limited direct competition reduces consumer alternatives, leading to less elastic demand and allowing firms to sustain higher markups.

5.4 Productivity

Firm productivity z_{ist} is composed of a permanent component z_i^P and a transitory component z_{ist}^T, given by:

$$z_{ist} = z_i^P \exp(z_{ist}^T) \tag{43}$$

The permanent component is independent of the sector in which the firm operates, capturing intrinsic differences in technical efficiency, such as managerial ability, technological know-how, or other persistent firm characteristics, and is assumed to be drawn from a Pareto distribution with shape parameter φ and lower bound normalized to 1:

$$F(z_i^P) = 1 - \left(\frac{1}{z_i^P}\right)^{\varphi} \qquad z_i^P \geq 1 \tag{44}$$

The transitory component is stochastic and sector-specific, evolving according to an AR(1) Markov process:

$$z_{ist}^T = \rho_s z_{ist-1}^T + \varepsilon_{ist}^T \qquad \varepsilon_{ist}^T \sim N(0, \sigma_s^T) \tag{45}$$

where both ρ_s, the persistence of the process, and σ_s, the standard deviation of idiosyncratic productivity shocks ε_{ist}^T, are sector-specific. Similar to Bianchi (2011), the persistence and volatility of the stochastic process vary across sectors, reflecting differences in exposure to risk. The tradable sector, for example, may experience greater volatility due to external shocks such as exchange rate fluctuations and global demand shifts, while the nontradable sector may face less volatile fluctuations. For example, Tornell and Westermann (2002) emphasizes the role of risky currency mismatches (i.e., firms borrowing in foreign currency but earning revenues in domestic currency, leaving them exposed to exchange rate risk) in amplifying external shocks.

5.5 Borrowing Constraints

Firms face a borrowing constraint that limits their ability to finance investment through debt. This constraint arises from frictions such as imperfect enforceability of contracts or asymmetric information between borrowers and lenders. In particular, I assume that firms can only borrow up to a fraction χ_s of their capital stock:

$$b_{ist} \leq \chi_s k_{ist} \tag{46}$$

where b_{ist} is the firm's debt, k_{ist} is its capital stock, and $\chi_s \geq 0$ is a sector-specific limit governing the tightness of the borrowing constraint. A higher χ_s allows firms to access more debt relative to their capital stock.

Borrowing constraints may vary across sectors due to differences in asset tangibility and collateralizability. Sectors that rely on tangible assets or produce collateralizable goods (e.g., manufacturing) may have higher borrowing limits, as their capital can be used as collateral for loans. In contrast, sectors dependent on intangible assets or service provision (e.g., professional services) may face tighter borrowing constraints since their capital is harder to pledge as collateral. Relatedly, Müller and Verner (2024) find that firms in the non-tradable sector are more reliant on debt secured by real estate collateral compared to firms in the tradable sector, suggesting that borrowing constraints are more binding in the non-tradable sector.

5.6 Capital Adjustment Costs

Firms incur capital adjustment costs when adjusting their capital stock over time, due to factors such as installation costs or technological rigidities. I assume that capital adjustment costs are quadratic in investment, imposing a penalty on abrupt changes in capital. Specifically, firms face an adjustment cost given by:

$$\Phi(k_{ist}, k_{ist-1}) = \frac{\phi_s}{2} \left(\frac{k_{ist} - (1 - \delta) k_{ist-1}}{k_{ist-1}} \right)^2 k_{ist-1}, \tag{47}$$

where $\phi_s \geq 0$ is a sector-specific adjustment cost parameter, and k_{ist} and k_{ist-1} denote the firm's current and previous-period capital stock, respectively. The quadratic structure introduces a convex friction, discouraging rapid capital accumulation or decumulation. The magnitude of ϕ_s determines the severity of this friction, with higher values implying greater investment rigidity.

Capital adjustment costs may vary across sectors due to differences in asset specificity. Sectors that rely on highly specialized capital (e.g., manufacturing, energy) may face higher capital adjustment costs because capital in these industries (e.g., customized machinery) is difficult to repurpose or resell. In contrast, sectors that use more general-purpose capital (e.g., retail, services) may have lower adjustment costs since capital (e.g., office spaces) can be more easily resold or redeployed across different uses.

5.7 Sectoral Mobility Costs

Firms face sectoral mobility costs when transitioning between the tradable and non-tradable sectors. These costs capture frictions that arise from capital irreversibility, industry-specific knowledge requirements, ownership restructuring (mergers and acquisitions), or regulatory barriers. As a result, firms cannot instantaneously or costlessly reallocate resources between sectors.

I model sectoral mobility costs as a one-time sunk cost $\Omega_{s,s'}$ that firms must pay when switching sectors. Specifically, if a firm chooses to transition from sector s to sector s', it incurs a cost:

$$\Omega_{s,s'}(k_{ist}) = \mathbb{I}_{s' \neq s} \omega_s k_{ist} \tag{48}$$

where $\mathbb{I}_{s' \neq s}$ is an indicator function that takes the value of one if $s' \neq s$ and 0 otherwise, and $\omega_s \geq 0$ is a sector-specific mobility parameter that scales with the firm's current-period capital stock k_{ist}. By tying mobility costs to capital stock, this formulation captures the idea that larger firms face proportionally higher costs when switching sectors. Moreover, mobility costs create hysteresis in sectoral choices, discouraging firms from switching sectors due to temporary shocks and requiring sufficient gains to justify transitioning.

5.8 Budget Constraint

Each period, the firm allocates funds to consumption, investment, debt repayment, capital adjustment costs, and potential sectoral mobility costs. The budget constraint is given by:

$$c_{ist} + k_{ist+1} - (1-\delta)k_{ist} + (1+r_t)b_{ist} + \Phi(k_{ist+1}, k_{ist}) + \Omega_{s,s'}(k_{ist}) = \pi_{ist} + b_{ist+1} \tag{49}$$

where the left-hand side represents the firm's total expenditures. The entrepreneur chooses consumption c_{ist}, invests in new capital $k_{ist+1} - (1-\delta)k_{ist}$, and pays down outstanding debt $(1+r_t)b_{ist}$. The firm also incurs capital adjustment costs $\Phi(k_{ist+1}, k_{ist})$ when modifying its capital stock and sectoral mobility costs $\Omega_{s,s\prime}(k_{ist})$ if it chooses to switch sectors. The right-hand side represents the firm's available resources, which include operating profits π_{ist} and new debt issuance b_{ist+1}.

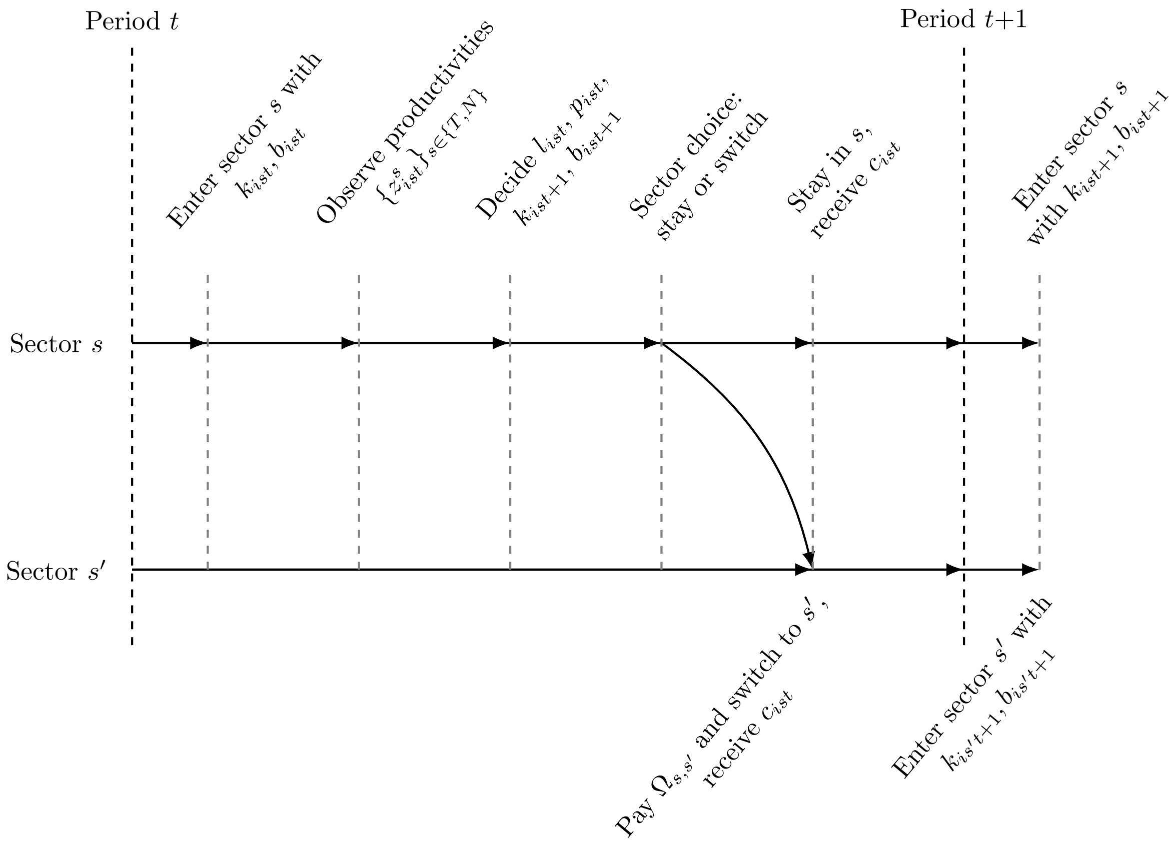
5.9 Timing Protocol

At the beginning of each period t, each firm i in sector s enters with a predetermined capital stock k_{ist} and outstanding debt b_{ist} from the previous period. It then observes its current sector-specific idiosyncratic productivity realizations $\{z^T_{ist}\}_{s\in\{T,N\}}$ which determine its profitability for the period. Given its current states, the firm hires labor l_{ist}, sets the price p_{ist} for its differentiated variety, chooses next period's debt b_{ist+1} and capital stock k_{ist+1} subject to capital adjustment costs $\Phi(k_{ist+1}, k_{ist})$.

Before observing next period's sector-specific productivity realizations $\{z^T_{ist+1}\}_{s\in\{T,N\}}$, the firm decides whether to remain in its current sector or switch to the alternative sector. If the firm switches sectors, it incurs the sectoral mobility cost $\Omega_{s,s'}(k_{ist})$. Finally, the firm owner enjoys consumption c_{ist}.

At the start of period $t+1$, the firm enters sector s' chosen in the previous period with its newly chosen capital stock k_{ist+1} and debt b_{ist+1} and then observes its new sector-specific productivity realizations. It then repeats the decision process.

Figure 5. Timing Protocol



5.10 Recursive Formulation

We can now express the firm's problem in recursive form. Let a_{ist} denote the firm's net worth, defined as $a_{ist} = k_{ist} - b_{ist} \geq 0$, that is, the firm's capital stock minus its outstanding debt. Primes are used to indicate next-period variables, and subscripts $s \in \{T, N\}$ denote sector-specific variables. The vector of endogenous state variables is given by $\mathbf{x} = (s, a, k)$, where s represents the firm's sector, a is net worth, and k is capital. The vector of exogenous idiosyncratic states, which captures firm productivity, is defined as $\mathbf{z} = (z^P, z^T_T, z^T_N)$. The vector of aggregate states, which includes the wage and interest rate, is given by $\mathbf{\Gamma} = (w, r)$. The firm's problem then is given by:

$$V(\mathbf{x}, \mathbf{z}, \mathbf{\Gamma}) = \max_{s', c, a', k', l, p} \left\{ \frac{c^{1-\gamma} - 1}{1-\gamma} + \beta \mathbb{E}_{\mathbf{z}'} V(\mathbf{x}', \mathbf{z}', \mathbf{\Gamma}') \right\} \tag{50}$$

subject to the budget constraint:

$$c + a' + \Phi_s(k', k) + \Omega_{s',s}(k) = p_s y_s - wl - (r+\delta)k - \kappa_s + (1+r)a \tag{51}$$

where $\Phi_s(k,k') = \frac{\phi_s(k'-(1-\delta)k)^2}{2k}$, $\Omega(k) = \mathbb{I}_{s' \neq s}\omega_s k$, $p_s = y_s^{-\frac{1}{\theta_s}}$, and $y_s = z^P \exp(z_s^T) k^{\alpha_s} l^{1-\alpha_s}$. The firm is also subject to the borrowing constraint $k' \leq \lambda_s a'$, with $\lambda_s = \frac{1}{1-\chi_s}$, as well as the non-negative net worth constraint $a' \geq 0$. Moreover, optimal labor demand is given by:[2]

$$l = \left(\frac{(1-\alpha_s)(\theta_s - 1)}{w\theta_s} \right)^{\frac{\theta_s}{1+\alpha_s(\theta_s-1)}} \left(z^P \exp(z_s^T) \right)^{\frac{\theta_s - 1}{1+\alpha_s(\theta_s-1)}} k^{\frac{\alpha_s(\theta_s-1)}{1+\alpha_s(\theta_s-1)}} \tag{52}$$

5.11 Stationary Equilibrium

A stationary partial equilibrium in this economy consists of a value function $V(\mathbf{x}, \mathbf{z}, \mathbf{\Gamma})$, policy functions $\sigma(\mathbf{x}, \mathbf{z}, \mathbf{\Gamma}) = (s'(\cdot),\ c(\cdot),\ a'(\cdot),\ k'(\cdot),\ l(\cdot),\ p(\cdot))$, and a stationary distribution of firms $\mu(\mathbf{x}, \mathbf{z})$ such that:

i. Given $\mathbf{\Gamma} = (w, r)$, the value function $V(\mathbf{x}, \mathbf{z}, \mathbf{\Gamma})$ satisfies the Bellman equation, and the policy functions $\sigma(\mathbf{x}, \mathbf{z}, \mathbf{\Gamma})$ solve the firm's dynamic problem subject to the budget constraint, borrowing constraint, and non-negative net worth constraint.

ii. The distribution $\mu(\mathbf{x}, \mathbf{z})$ is invariant over time and satisfies the law of motion:

$$\mu'(\mathbf{x}', \mathbf{z}') = \mathcal{T}(\mu(\mathbf{x}, \mathbf{z})) \tag{53}$$

where $\mathcal{T}(\cdot)$ is the operator that maps the current distribution into the next period's distribution, given firms' optimal decision rules and the exogenous shock processes.

[2] See proof in Appendix Appendix B.1

6 Quantitative Exercise

This section outlines the simulated method of moments (SMM) approach used to calibrate the model. The quantified model is then used to evaluate the role of sectoral asymmetries in accounting for the reallocation of economic activity and the associated TFP losses observed in Spain following the decline in interest rates after the Maastricht Treaty, as documented in Section 2.

6.1 Calibration Strategy

A period in the model corresponds to one year. To calibrate the model, I begin by partitioning the parameter space into two groups. The first group includes parameters that are predetermined based on standard values in the literature. I set the discount factor to $\beta = 0.875$, the coefficient of relative risk aversion to $\gamma = 1.5$, and the depreciation rate to $\delta = 0.06$. The elasticity of substitution between tradable and nontradable goods is $\eta = 1.5$, and the shape parameter of the Pareto distribution that governs the permanent component of firm productivity is $\varphi = 30$. Finally, I normalize both the wage rate w_t and the aggregate price level P_t to one throughout.

The remaining parameters are calibrated through simulated method of moments (SMM). Let Θ denote the vector of parameters to be calibrated internally. Given the high dimensionality of the parameter space, I discretize Θ over a grid and conduct a grid search. For each candidate vector Θ, I simulate the model and compute the time paths of a set of J target variables, indexed by $j = 1, \ldots, J$, over T time periods, indexed by $t = 1, \ldots, T$. Let $\mathcal{M}_t^j(\Theta)$ denote the model-generated value of variable j at time t under parameter vector Θ, and $\mathcal{D}_t^j$ the corresponding empirical value. The optimal parameter vector Θ^* minimizes the squared distance between the model and the data, as captured by the following objective function:

$$\Theta^* = \arg\min_{\Theta} \; M(\Theta) W M(\Theta)' \tag{54}$$

where $M(\Theta)$ is the vector of moment errors that stacks the deviations between model-implied and empirical paths across all variables and time periods, defined as:

$$M(\Theta) = \begin{bmatrix} \mathcal{M}_1^1(\Theta) - \mathcal{D}_1^1 \\ \mathcal{M}_2^1(\Theta) - \mathcal{D}_2^1 \\ \vdots \\ \mathcal{M}_T^1(\Theta) - \mathcal{D}_T^1 \\ \mathcal{M}_1^2(\Theta) - \mathcal{D}_1^2 \\ \vdots \\ \mathcal{M}_T^J(\Theta) - \mathcal{D}_T^J \end{bmatrix} \tag{55}$$

and W is a weighting matrix, which I set equal to the identity matrix.

To evaluate each candidate parameter vector Θ, I first compute the stationary equilibrium of the economy under the prevailing real interest rate in 1992, as proxied by the inflation-adjusted interest rate on long-term government debt. The stationary equilibrium is computed in partial equilibrium, as described in Section 5.11, where firms are optimizing subject to idiosyncratic shocks, but aggregates are constant and the distribution of firms over states is invariant. I then simulate the model's transition dynamics in response to the observed sequence of real interest rates from 1992 to 2008 in order to generate model-implied time series $\mathcal{M}_t^j(\Theta)$, which are compared to their empirical counterparts $\mathcal{D}_t^j$ over this period. The target variables are the log changes relative to 1992 in TFP, tradable output, and nontradable output.

6.2 Transition Experiment

Table 1 summarizes the internally calibrated parameters, as well as the model's overall fit. The differences reported in Panel B are presented in relative terms, calculated as the absolute difference divided by the average of the two sectoral values. Figure 6 compares the model-implied dynamics to the corresponding empirical patterns using the parameter set that achieves the best overall fit.

The model assigns a modest difference in demand elasticity across sectors, with the tradable sector exhibiting a 10% higher elasticity than the nontradable sector. The capital share (α) is estimated to be about 16% higher in the tradable sector, and fixed operating costs (κ) are positive for tradables but zero for nontradables, consistent with

Table 1. Fitted Parameters by Sector and Model Fit

Panel A: **Assigned Parameters**			
	Parameter	Value	
	β	0.875	
	γ	1.50	
	δ	0.06	
	η	1.50	
	φ	30.0	
Panel B: **Fitted Parameters**			
Parameter	Tradable (T)	Nontradable (N)	% Diff.
θ_s	4.15	3.75	10%
α_s	0.34	0.29	16%
κ_s	0.11	0.00	200%
ρ_s	0.53	0.62	16%
σ_s	0.28	0.36	25%
λ_s	1.12	1.02	9%
ϕ_s	4.51	2.62	53%
ω_s	0.14	0.08	55%
Panel C: **Model Fit (RMSE)**			
	TFP	0.006	
	Y^T	0.118	
	Y^N	0.717	
	Total	0.420	

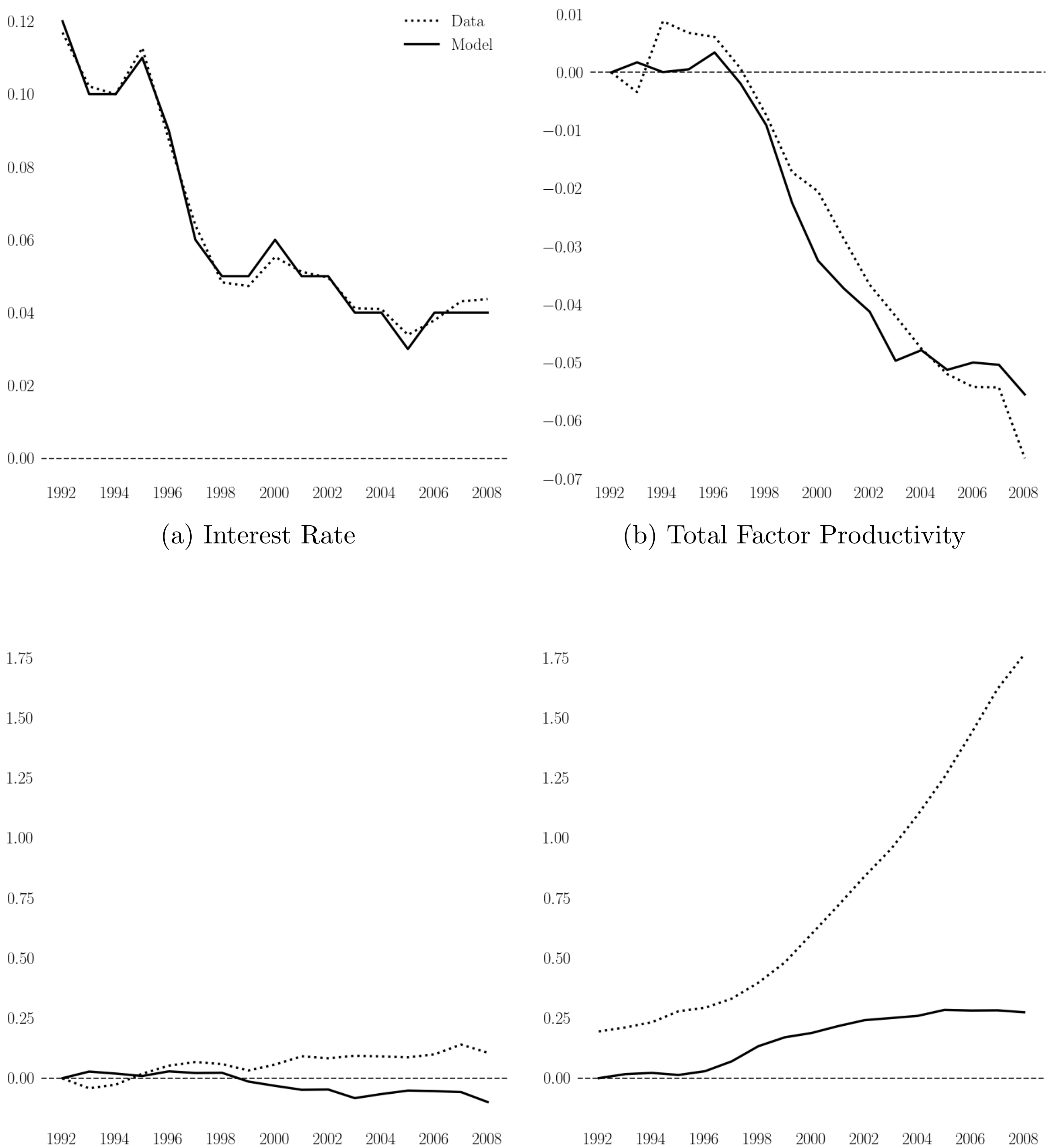
higher scale requirements and capital intensity in tradable production.

Turning to the parameters governing firm-level productivity dynamics, the model estimates that the tradable sector exhibits 16% lower persistence (ρ) and 25% higher volatility (σ) of productivity shocks relative to the nontradable sector, suggesting a riskier operating environment for tradable firms. Borrowing constraints (λ) are tighter in the nontradable sector, although the difference is relatively modest, at 9%.

Finally, the model assigns an important role to capital frictions. Capital adjustment costs (ϕ) are estimated to be roughly 50% higher in the tradable sector. In addition, sectoral switching costs (ω) are found to be quantitatively significant and comparable in magnitude to capital adjustment costs, underscoring the importance of frictions in reallocating capital across sectors. Intuitively, these two frictions dampen the tradable sector's response to declining interest rates along both the intensive and extensive margins: capital adjustment costs restrict the ability of existing tradable firms to scale up investment, while switching costs inhibit firms in the nontradable sector from entering tradables when capital becomes cheaper, further limiting the tradable sector's expansion.

As illustrated in Figure 6, given the observed sequence of interest rates during this period, the model performs well in replicating the aggregate TFP loss, with a root mean squared error of 0.006, as well as the reallocation of economic activity away from the tradable sector. However, the model falls short in accounting for the full extent of the expansion in the nontradable sector. While nontradable output increased by 176% relative to trend in the data, the model accounts for only 27% of the rise. This discrepancy suggests that additional mechanisms beyond the frictions and sectoral asymmetries captured in the model may have contributed to the observed boom in nontradable output.

Figure 6. Transition Dynamics: Model vs. Data



(a) Interest Rate

(b) Total Factor Productivity

(c) Tradable Output

(d) Non-Tradable Output

Notes: Each panel compares the simulated transition dynamics (solid line) to the corresponding empirical series for Spain (dashed line).

7 Conclusion

This paper examines the channels through which capital inflows may reduce aggregate productivity by inducing changes in the composition of output between tradable and nontradable sectors. Following the Maastricht Treaty, I document a reallocation of activity from tradable to nontradable sectors—more pronounced in peripheral economies—and a divergence in TFP dynamics. To interpret these patterns, I extend the canonical misallocation framework to a multi-sector setting and develop a quantitative model with rich firm- and sector-level heterogeneity. The calibrated model attributes a central role to capital frictions—both within sectors and across sectors—in accounting for the observed dynamics, relative to other sectoral characteristics and distortions. These findings suggest that policies facilitating capital reallocation, such as improving secondary markets for capital goods, streamlining business transfers, and lowering barriers to sectoral mobility, can play an important role in improving resource allocation and aggregate productivity.

References

Bedendo, M. and Colla, P. (2015). Sovereign and corporate credit risk: Evidence from the Eurozone. *Journal of corporate Finance*, 33:34–52.

Benigno, G., Converse, N., and Fornaro, L. (2015). Large capital inflows, sectoral allocation, and economic performance. *Journal of International Money and Finance*, 55:60–87.

Benigno, G. and Fornaro, L. (2014). The financial resource curse. *The Scandinavian Journal of Economics*, 116(1):58–86.

Benigno, G., Fornaro, L., and Wolf, M. (2025). The global financial resource curse. *American Economic Review*, 115(1):220–262.

Bevilaqua, J., Hale, G. B., and Tallman, E. (2020). Corporate yields and sovereign yields. *Journal of International Economics*, 124:103304.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review*, 101(7):3400–3426.

Brunnermeier, M. K. and Reis, R. (2023). *A crash course on crises: macroeconomic concepts for run-ups, collapses, and recoveries.* Princeton University Press.

Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and Development: A Tale of Two Sectors. *The American Economic Review*, 101(5).

Cette, G., Fernald, J., and Mojon, B. (2016). The pre-Great Recession slowdown in productivity. *European Economic Review*, 88:3–20.

Dias, D. A., Marques, C. R., and Richmond, C. (2016). Misallocation and productivity in the lead up to the Eurozone crisis. *Journal of Macroeconomics*, 49:46–70.

Dias, D. A., Robalo Marques, C., and Richmond, C. (2020). A tale of two sectors: why is misallocation higher in services than in manufacturing? *Review of Income and Wealth*, 66(2):361–393.

Durbin, E. and Ng, D. (2005). The sovereign ceiling and emerging market corporate bond spreads. *Journal of international Money and Finance*, 24(4):631–649.

Eichengreen, B. and Mody, A. (2000). What Explains Changing Spreads on Emerging Market Debt? In *Capital Flows and the Emerging Economies: Theory, Evidence, and Controversies*, pages 107–134. University of Chicago Press.

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The next generation of the Penn World Table. *American economic review*, 105(10):3150–3182.

García-Santana, M., Moral-Benito, E., Pijoan-Mas, J., and Ramos, R. (2020). Growing like Spain: 1995–2007. *International Economic Review*, 61(1):383–416.

Gopinath, G., Kalemli-Özcan, , Karabarbounis, L., and Villegas-Sanchez, C. (2017). Capital Allocation and Productivity in South Europe. *The Quarterly Journal of Economics*, 132(4).

Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing TFP in China and India. *Quarterly Journal of Economics*, 124(4):1403–1448.

Kalantzis, Y. (2015). Financial fragility in small open economies: firm balance sheets and the sectoral structure. *Review of Economic Studies*, 82(3):1194–1222.

Müller, K. and Verner, E. (2024). Credit allocation and macroeconomic fluctuations. *Review of Economic Studies*, 91(6):3645–3676.

O'Mahony, M. and Timmer, M. P. (2009). Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database. *The Economic Journal*, 119(538):F374–F403.

Ozhan, G. K. (2020). Financial intermediation, resource allocation, and macroeconomic interdependence. *Journal of Monetary Economics*, 115:265–278.

Reis, R. (2013). The Portuguese Slump and Crash and the Euro-Crisis. *Brookings Papers on Economic Activity*, (1):1–52.

Schneider, M. and Tornell, A. (2004). Balance sheet effects, bailout guarantees and financial crises. *Review of Economic Studies*, 71(3):883–913.

Tornell, A. and Westermann, F. (2002). Boom-bust cycles in middle income countries: Facts and explanation. *IMF Staff Papers*, 49(Suppl 1):111–155.

Valentinyi, A. and Herrendorf, B. (2008). Measuring factor income shares at the sectoral level. *Review of Economic Dynamics*, 11(4):820–835.

Appendix A Proofs from the Illustrative Model

Theorem Appendix A.1. *The optimal capital choice that maximizes expected profit is given by:*

$$k^* = \frac{z^{\theta_s - 1}}{\mu_s^{\theta_s} r^{\theta_s} \mathbb{E}(1+\tilde{\tau}^k)^{\theta_s}} \tag{56}$$

Proof. The firm solves the following maximization problem:

$$\max_k \mathbb{E}\left[(zk)^{\frac{\theta_s - 1}{\theta_s}} - (1+\tilde{\tau}^k) r k\right] \tag{57}$$

The first-order condition with respect to k is:

$$\frac{\theta_s - 1}{\theta_s} z^{\frac{\theta_s - 1}{\theta_s}} k^{-\frac{1}{\theta_s}} = r\mathbb{E}(1+\tilde{\tau}^k) \tag{58}$$

Rearranging for k, we obtain the optimal capital choice:

$$k^* = \frac{z^{\theta_s - 1}}{\mu_s^{\theta_s} r^{\theta_s} \mathbb{E}(1+\tilde{\tau}^k)^{\theta_s}} \tag{59}$$

where $\mu_s = \frac{\theta_s}{\theta_s - 1}$. □

Theorem Appendix A.2. *The expected capital distortion is given by:*

$$\mathbb{E}(1+\tilde{\tau}^k) = 1 + \mathbb{E}(\tau_i^k) + \mathbb{E}(\tau_s^k) + \mathbb{E}(\tau_i^k)\mathbb{E}(\tau_s^k) + Cov(\tau_i^k, \tau_s^k) \tag{60}$$

Proof. Expanding the expectation of the composite distortion:

$$\mathbb{E}(1+\tilde{\tau}^k) = \mathbb{E}[(1+\tau_i^k)(1+\tau_s^k)] \tag{61}$$

$$= 1 + \mathbb{E}(\tau_i^k) + \mathbb{E}(\tau_s^k) + \mathbb{E}(\tau_i^k \tau_s^k) \tag{62}$$

By the definition of covariance:

$$\mathrm{Cov}(\tau_i^k, \tau_s^k) = \mathbb{E}(\tau_i^k \tau_s^k) - \mathbb{E}(\tau_i^k)\mathbb{E}(\tau_s^k) \tag{63}$$

Substituting $\mathbb{E}(\tau_i^k \tau_s^k) = \mathrm{Cov}(\tau_i^k, \tau_s^k) + \mathbb{E}(\tau_i^k)\mathbb{E}(\tau_s^k)$ into the previous equation:

$$\mathbb{E}(1 + \tilde{\tau}^k) = 1 + \mathbb{E}(\tau_i^k) + \mathbb{E}(\tau_s^k) + \mathbb{E}(\tau_i^k)\mathbb{E}(\tau_s^k) + \mathrm{Cov}(\tau_i^k, \tau_s^k) \tag{64}$$

□

Theorem Appendix A.3. *The efficient level of capital in the absence of distortions is given by:*

$$k^e = \frac{z^{\theta_s - 1}}{\mu_s^{\theta_s} r^{\theta_s}} \tag{65}$$

Proof. Absent capital wedges, the firm chooses its capital k to maximize expected profit under an isoelastic production function:

$$\pi(s) = \max_k \ \mathbb{E}\left[(zk)^{\frac{\theta_s - 1}{\theta_s}} - rk\right] \tag{66}$$

Taking the first-order condition (FOC) with respect to k:

$$\frac{\theta_s - 1}{\theta_s} z^{\frac{\theta_s - 1}{\theta_s}} k^{-\frac{1}{\theta_s}} = r \tag{67}$$

Solving for k, we isolate the efficient capital level:

$$k^e = \frac{z^{\theta_s - 1}}{\mu_s^{\theta_s} r^{\theta_s}} \tag{68}$$

where $\mu_s = \frac{\theta_s}{\theta_s - 1}$. □

Theorem Appendix A.4. *Distortions lead to a greater reduction in capital accumulation in sectors with a higher elasticity of substitution.*

Proof. By definition, the ratio between the optimal capital choice and its efficient level is given by:

$$\frac{k^*}{k^e} = \left(\frac{1}{\mathbb{E}(1+\tilde{\tau}^k)}\right)^{\theta_s} \tag{69}$$

Taking the derivative with respect to θ_s, we apply the logarithmic differentiation rule:

$$\frac{d}{d\theta_s}\left(\frac{1}{\mathbb{E}(1+\tilde{\tau}^k)}\right)^{\theta_s} = \left(\frac{1}{\mathbb{E}(1+\tilde{\tau}^k)}\right)^{\theta_s} \log\left(\frac{1}{\mathbb{E}(1+\tilde{\tau}^k)}\right) \tag{70}$$

Simplifying:

$$\frac{d(k^*/k^e)}{d\theta_s} = -\left(\mathbb{E}(1+\tilde{\tau}^k)\right)^{-\theta_s} \log\left(\mathbb{E}(1+\tilde{\tau}^k)\right) \tag{71}$$

Since $\mathbb{E}(1+\tilde{\tau}^k) > 1$ for $\tilde{\tau}^k > 0$, it follows that $\log\left(\mathbb{E}(1+\tilde{\tau}^k)\right)$ is positive (since the logarithm of a number greater than one is positive), $\left(\mathbb{E}(1+\tilde{\tau}^k)\right)^{-\theta_s}$ is also positive (since it is an exponentiation of a positive quantity), and thus the overall derivative is negative, implying that as θ_s increases, k^*/k^e decreases.

Because k^*/k^e is less than one for positive capital wedges, a decrease in this ratio implies that distortions push the capital choice further away from the efficient level. This means that capital distortions have a stronger effect in high-elasticity sectors. □

Theorem Appendix A.5. *Given the optimal capital choice k^*, the expected profit of a firm operating in sector s, subject to capital distortions τ^k, is:*

$$\pi^*(s) = \frac{(\theta_s - 1)^{\theta_s - 1} z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s - 1} \mathbb{E}(1+\tilde{\tau}^k)^{\theta_s - 1}} \tag{72}$$

Proof. The firm maximizes its expected profit:

$$\pi^*(s) = \max_k \; \mathbb{E}\left[(zk)^{\frac{\theta_s - 1}{\theta_s}} - (1+\tilde{\tau}^k)rk\right] \tag{73}$$

Rewriting:

$$\pi^*(s) = z^{\frac{\theta_s - 1}{\theta_s}} k^{\frac{\theta_s - 1}{\theta_s}} - \mathbb{E}(1+\tilde{\tau}^k)rk \tag{74}$$

Factoring out k:

$$\pi^*(s) = \left[z^{\frac{\theta_s - 1}{\theta_s}} k^{-\frac{1}{\theta_s}} - \mathbb{E}(1 + \tilde{\tau}^k) r \right] k \tag{75}$$

Substituting the optimal capital k^*

$$k^* = \frac{z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s}} \tag{76}$$

into the profit function:

$$\pi^*(s) = \left[z^{\frac{\theta_s - 1}{\theta_s}} \left(\frac{z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s}} \right)^{-\frac{1}{\theta_s}} - \mathbb{E}(1 + \tilde{\tau}^k) r \right] k^* \tag{77}$$

Computing k^* raised to the power $-\frac{1}{\theta_s}$:

$$k^{-\frac{1}{\theta_s}} = \left(\frac{z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s}} \right)^{-\frac{1}{\theta_s}} \tag{78}$$

Substituting:

$$\pi^*(s) = \big((\theta_s - 1) r \mathbb{E}(1 + \tilde{\tau}^k) \big) \left(\frac{z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s}} \right) \tag{79}$$

Simplifying:

$$\pi^*(s) = \frac{(\theta_s - 1)^{\theta_s - 1} z^{\theta_s - 1}}{\theta_s^{\theta_s} r^{\theta_s - 1} \mathbb{E}(1 + \tilde{\tau}^k)^{\theta_s - 1}} \tag{80}$$

□

Theorem Appendix A.6. *A firm choosing between two sectors, A and B, which differ in their elasticity of substitution, with $\theta_A > \theta_B > 1$, strictly prefers to operate in sector A over sector B if its productivity z satisfies:*

$$z > r \left(\frac{\theta_B - 1}{\theta_A - 1} \frac{\mu_A^{\theta_A}}{\mu_B^{\theta_B}} \right)^{\frac{1}{\theta_A - \theta_B}} \frac{\mathbb{E}(1 + \tilde{\tau}^k(A))^{\frac{\theta_A - 1}{\theta_A - \theta_B}}}{\mathbb{E}(1 + \tilde{\tau}^k(B))^{\frac{\theta_B - 1}{\theta_A - \theta_B}}} \tag{81}$$

Proof. The firm prefers sector A over B if:

$$\pi^*(A) > \pi^*(B) \tag{82}$$

Plugging in the expressions:

$$\frac{(\theta_A - 1)^{\theta_A - 1} z^{\theta_A - 1}}{\theta_A^{\theta_A} r^{\theta_A - 1} \mathbb{E}(1 + \tilde{\tau}^k(A))^{\theta_A - 1}} > \frac{(\theta_B - 1)^{\theta_B - 1} z^{\theta_B - 1}}{\theta_B^{\theta_B} r^{\theta_B - 1} \mathbb{E}(1 + \tilde{\tau}^k(B))^{\theta_B - 1}} \tag{83}$$

Rewriting:

$$z^{\theta_A - \theta_B} > r^{\theta_A - \theta_B} \left(\frac{\theta_A^{\theta_A} (\theta_B - 1)^{\theta_B - 1}}{\theta_B^{\theta_B} (\theta_A - 1)^{\theta_A - 1}} \right) \frac{\mathbb{E}(1 + \tilde{\tau}^k(A))^{\theta_A - 1}}{\mathbb{E}(1 + \tilde{\tau}^k(B))^{\theta_B - 1}} \tag{84}$$

Given that the sectoral markup is given by $\mu_s = \frac{\theta_s}{\theta_s - 1}$, we obtain the threshold condition:

$$z > r \left(\frac{\theta_B - 1}{\theta_A - 1} \frac{\mu_A^{\theta_A}}{\mu_B^{\theta_B}} \right)^{\frac{1}{\theta_A - \theta_B}} \frac{\mathbb{E}(1 + \tilde{\tau}^k(A))^{\frac{\theta_A - 1}{\theta_A - \theta_B}}}{\mathbb{E}(1 + \tilde{\tau}^k(B))^{\frac{\theta_B - 1}{\theta_A - \theta_B}}} \tag{85}$$

□

Appendix B Proofs from the Quantitative Model

Theorem Appendix B.1. *For any given vector state* $(\mathbf{x}, \mathbf{y}, \mathbf{\Gamma}) = (s, a, k, z^P, z^T_T, z^T_N, w, r)$, *the firm's optimal labor demand is given by:*

$$l = \left(\frac{(1-\alpha_s)(\theta_s - 1)}{w\theta_s} \right)^{\frac{\theta_s}{1+\alpha_s(\theta_s-1)}} \left(z^P \exp(z^T_s) \right)^{\frac{\theta_s - 1}{1+\alpha_s(\theta_s-1)}} k^{\frac{\alpha_s(\theta_s-1)}{1+\alpha_s(\theta_s-1)}} \tag{86}$$

Proof. Given that the firm's output is $y_s = z^P \exp(z^T_s) k^{\alpha_s} l^{1-\alpha_s}$ and the inverse demand function for its variety is $p_s = y_s^{-\frac{1}{\theta_s}}$, it follows that the firm's total revenue is $p_s y_s = y_s^{\frac{\theta_s - 1}{\theta_s}}$ and its operating profits are:

$$\pi_s = p_s y_s - wl - \kappa_s = \left(z^P \exp(z^T_s) k^{\alpha_s} l^{1-\alpha_s} \right)^{\frac{\theta_s-1}{\theta_s}} - wl - \kappa_s \tag{87}$$

Differentiating with respect to l:

$$\frac{\partial \pi_s}{\partial l} = \frac{\theta_s - 1}{\theta_s} \left(z^P \exp(z^T_s) k^{\alpha_s} l^{1-\alpha_s} \right)^{\frac{\theta_s-1}{\theta_s} - 1} (1-\alpha_s) z^P \exp(z^T_s) k^{\alpha_s} l^{-\alpha_s} - w \tag{88}$$

Setting the derivative equal to zero and solving for l:

$$l = \left(\frac{(1-\alpha_s)(\theta_s - 1)}{w\theta_s} \right)^{\frac{\theta_s}{1+\alpha_s(\theta_s-1)}} \left(z^P \exp(z^T_s) \right)^{\frac{\theta_s - 1}{1+\alpha_s(\theta_s-1)}} k^{\frac{\alpha_s(\theta_s-1)}{1+\alpha_s(\theta_s-1)}} \tag{89}$$

□